

# Capacity Market

## as means to avoid blackouts

Assessment of the impact of implementation of a comprehensive capacity market mechanism in Poland



## Appendix I



**PKEE**  
Polish Electricity  
Association

# The PKEE fundamental model

The role of the PKEE fundamental model is to solve the optimisation problem described by a set of mathematical equations and inequalities. The starting point of every optimisation problem is to define the goal function and direction of its optimisation, and then introduction of constraining equations.

Basic information about the model:

1. Model operating horizon scope: equal to the input data horizon
2. Optimisation calculation resolution: 1 hour
3. Goal of the optimisation exercise: minimisation of the goal function (system costs)
4. Type of optimisation programming used – LP (Linear Programming)

## Goal function

The scope of optimisation is minimisation of the goal function. The goal function  $Z$  (1.1) reflects the sum of total discounted system costs, i.e. variable costs, costs of activating the units, costs of reduction of electricity consumption, costs of (or revenues from) cross-border exchange and costs related to future investment decisions. The variable costs are calculated for each group of power plants, every day and over each hour. Generating units may be grouped in any way that is adopted – the starting point is separation of all centrally dispatched generation units (CDGUs) and grouping the nCDGU by generating technology (e.g. Gas cogeneration, Wind, Solar, etc.). The costs of activations are calculated for each group of power plants every day. All activations are performed from a cold start. Costs of the DSR (Demand Side Response) concern individual consumers on every day. The O&M and investment related costs are assumed constant over time and depending only on the power station concerned. Costs relating to cross-border exchange are calculated for every country participating in exchange every day.

$$\begin{aligned} \min Z = & \sum_{t, dataPL, ele} (Generation_{ele, dataPL, t} * c_{G_{ele, dataPL}} * \#_{days_{dataPL}}) \\ & + \sum_{dataPL, ele} (Activtn_{ele, dataPL} * c_{U_{ele, dataPL}} * \#_{days_{dataPL}}) \\ & + \sum_{kons, dataPL, t} (DSR_{kons, dataPL, t} * c_{DSR_{kons}} * \#_{days_{dataPL}}) \\ & + \sum_{ele} (Cpcty_{ele} * c_{O\&M_{ele}}) \\ & + \sum_{ct, t, dataPL} [Flows_{ct, dataPL, t} * (CBF_{Price_{ct, dataPL, t}})] \end{aligned} \quad (1.1)$$

$$\forall t \in T, \forall ele \in Ele, \forall dataPL \in Kal, \forall kons \in Kons, ct \in CT$$

Where:

$Z$  – goal function

$c_{G_{ele,dataPL}}$  – variable production cost in a specific power plant on a given day  
(= $DTable\_Gen\_Cost_{dataPL,ele}$ )

$c_{U_{ele,dataPL}}$  – cost of activation of a specific power plant on a given day  
(= $DTable\_CostActvtn_{dataPL,ele}$ )

$c_{DSR_{kons}}$  – cost of demand side reduction by a specific consumer  
(= $DTable\_DSR\_Price_{kons}$ )

$c_{O\&M_{ele}}$  – O&M cost at a specific power plant (=  $DTable\_OM_{ele}$ )

$Generation_{ele,dataPL,t}$  – total generation at specific power plant in a given hour

$DSR_{kons,dataPL,t}$  – value of demand side reduction for specific consumer on specific day on specific hour

$Activtn_{ele,dataPL}$  – activated gross capacities in power stations (MW)

$Cpcty_{ele}$  – capacity of specific power plant

$\#\_days_{dataPL}$  – days counter

$Flows_{ct,dataPL,t}$  – balance of cross-border trade with specific country on a given day in a given hour

$CBF\_Price_{ct,dataPL,t}$  – price of imported /exported unit of energy  
(= $DTable\_CBF\_Price_{ct,dataPL,t}$ )

$ele$  – set of all power stations,  $ele \in Ele$

$dataPL$  – set of representative days,  $dataPL \in Kal$

$t$  – time (hours),  $t \in T = \{1, \dots, 24\}$

$kons$  – consumers,  $kons \in Kons = \{1, \dots, 50\}$

$ct$  – country,  $ct \in CT$

## Constraining equations and inequalities

Besides the goal function every optimisation problem has a set of constraints. Constraints may result from mathematical nature of variables (e.g. positive value of generation), model logic (demand supply balance in the system), or from technical aspects (e.g. technical minimum of a generating unit, maximum attainable capacity).

## Electricity supply and demand balancing

The fundamental constraining equation in the process of optimisation of economic dispatch is the equality constraint enforcing equal values of the sum of generation and of demand in every moment in time. In the case under consideration the left side of the equation that defines supply includes not only production but also the DSR and balance of cross-border exchange. At the same time the right side of the equation defining the demand was extended with the sum of energy consumed for pumping in pumped-storage power stations. The equations balancing demand and supply (1.2) in the model are represented by a block of Power Balance objects (dataPL,t).

$$\begin{aligned} \sum_{t, dataPL, ele} (Generation_{ele, dataPL, t}) + \sum_{kons, dataPL, ele} (DSR_{kons, dataPL, t}) + Flows_{dataPL, t} \\ = Demand_{dataPL, t} + \sum_{esp, dataPL} (ESP\_Pumping_{esp, dataPL, t}) \end{aligned} \quad (1.2)$$

$$\forall dataPL \in Kal, \forall t \in T$$

Where:

$Demand_{dataPL, t}$  – value of electricity demand in the National Electricity System on a given day in a given hour (=DTable\_Demand<sub>dataPL, t</sub>)

$Flows_{dataPL, t}$  – cross-border balance on a given day in a given hour (=DTable\_Flows<sub>dataPL, t</sub>)

$ESP\_Pumping_{esp, dataPL, t}$  – value of energy consumed for pumping water in pumped-storage power stations on a given day in a given hour

## Technical constraints of power stations

Issues relating to capacity expansion in the electricity system require introduction of constraining equations. The installed capacity of a specific power plant in a given year is equal to the capacity defined in the input data, unless the capacity expansion algorithm has decided to reduce (disconnect) it or build new generating capacity.

Due to technical constraints on the power stations it is necessary to formulate capacity constraints relating to the conditions of operation of generating units. Inequality (1.3) enforces RES generation at a set level defined in goals concerning the share of renewable sources energy in the total demand. The sum of RES generation over the year must thus be greater or equal to the product of multiplication of demand by the percentage share adopted as the goal. In case of recalculation of the model for the needs of the main report this limitation may cause expansion of the RES capacity above the PEP2050 values, which are introduced as the minimum starting scenario. Generation from individual dispatchable generating units (1.4) may not be greater than their activated gross capacity less the capacity used for internal consumption and losses. Generation from non-dispatchable sources (1.5) is in turn defined as the product of multiplication of installed capacity and factor of average capacity utilisation in a given source in a given hour derived from hourly average profile, that is different for different weather scenario calculations. The  $Cpcty_{ele}$  value constituting the attainable capacity of a given unit is retrieved from the  $DTable\_Cpcty_{ele}$  table (1.6), while its subset  $Cpcty_{es}$  defining the attainable capacities of dispatchable units constitutes the upper constraint for the  $InOper_{es,dataPL}$  variable representing the activated gross capacities in dispatchable power stations (1.7). Capacity activated on a given day is in turn not greater than the sum of capacity activated on the preceding day and the capacity activated in the moment of time under consideration (1.10). The value of demand side reduction in a given hour is not greater than the value of capacity offered in the DSR by the given consumer (1.8). The constraint (1.9) forces generation in a given dispatchable unit in a given hour at the level greater or equal to the product of multiplication of in-service capacity less losses and the technical minimum coefficient of the unit. The equation (1.11) forces generation in dispatchable units at the level not lower than resulting from the set value of load coefficient of units operating in forced mode considering losses and internal consumption. Equation 1.12 includes also a constraint of production output limit in opencast lignite mines (OLM). The sum of energy supplied in form of fuel, resulting from lignite-fired units' generation may not exceed the opencast mine production output limit assumed in table  $DTable\_LimitOLM_{olm}$  expressed in energy units.

$$\sum_{eres,dataPL,t} (Generation_{eres,dataPL,t} * \#\_days_{dataPL}) \geq RES\_Target * \sum_{dataPL,t} (Demand_{dataPL,t} * \#\_days_{dataPL}) \quad (1.3)$$

$$\forall eres \in Ele, \forall dataPL \in Kal, \forall t \in$$

where:

$Generation_{eres,dataPL,t}$  – RES generated energy volume on a given day in given hour

$RES\_Target$  – share of RES electricity in domestic demand (=  $DTable\_RES\_Target$ )

$$\begin{aligned}
& Generation_{es,dataPL,t} \leq InOper_{es,dataPL} * (1 - DTable\_Losses_{dataPL,es}) * (1 - DTable\_OwnCons_{es}) \\
& \forall es \in Ele, \forall dataPL \in Kal, t \in T
\end{aligned} \tag{1.4}$$

$$\begin{aligned}
& Generation_{ens,dataPL,t} = Cpcty_{ens} * DTable\_Production\_ENS_{dataPL,t,ens} \\
& \forall ens \in Ele, \forall dataPL \in Kal, t \in T
\end{aligned} \tag{1.5}$$

$$Cpcty_{ele} = DTable\_Cpcty_{ele} \quad \forall ele \in Ele \tag{1.6}$$

$$InOper_{es,dataPL} \leq Cpcty_{es} \quad \forall es \in Ele, \forall dataPL \in Kal \tag{1.7}$$

$$DSR_{kons,dataPL,t} \leq DTable\_DSR\_Cpcty_{kons} \quad \forall kons \in Kons, \forall dataPL \in Kal, t \in T \tag{1.8}$$

$$\begin{aligned}
& \frac{Generation_{es,dataPL,t}}{1 - DTable\_OwnCons_{es} * DTable\_TechMin_{es}} \geq InOper_{es,dataPL} * (1 - DTable\_Losses_{dataPL,es}) * DTable\_TechMin_{es} \\
& \forall es \in Ele, \forall dataPL \in Kal
\end{aligned} \tag{1.9}$$

$$InOper_{es,dataPL} \leq InOper_{es,dataPL-1} + Activtn_{es,dataPL} \quad \forall es \in Ele, \forall dataPL \in Kal \tag{1.10}$$

$$\begin{aligned}
& Generation_{es,dataPL,t} \geq DTable_{LF_{es}} * Cpcty_{es} \\
& * (1 - DTable\_OwnCons_{es}) * (1 - DTable\_Losses_{dataPL,es}) \\
& \forall es \in Ele, \forall dataPL \in Kal, t \in T
\end{aligned} \tag{1.11}$$

$$\begin{aligned}
& \sum_{es,dataPL,t, \text{map\_olm\_ele}} \frac{\#\_dni_{dataPL} * Generation_{es,dataPL,t} * 3.6}{DTable\_Efficiency_{es} * 1000} \leq DTable\_LimitOLM_{olm} \\
& \forall es \in Ele, \forall dataPL \in Kal, t \in T, \forall olm \in OLM
\end{aligned} \tag{1.12}$$

The next set of constraints concerns operation of peaking pumped-storage power plants. The basic constraint is the water balance defined as the water balance in the hour preceding the hour considered plus the water head balance in the current hour resulting from the difference between pumping and generation (1.13). When hour '1' is being considered the balance in preceding hour is calculated as the balance for the last hour of the preceding day. If day '1' is being considered then the calculations are performed for the last hour of last day, meaning that every year is self-balanced. The sum of energy consumed for pumping and energy generated in a given hour may not exceed the value of capacity resulting from pumping ability in a peaking pumped-storage power plant (a product of pumping capacity to generation capacity and net dispatchable capacity) (1.14). In addition the water balance is subject to constraint resulting from the reservoir capacity (1.15).

$$ESP\_Balance_{esp,dataPL,t} = ESP\_Balance_{esp,dataPL,t-1(t>1)} + ESP\_Balance_{esp,dataPL-1,24(t=1,dataPL>1)} \\ + ESP\_Balance_{esp,d6,"24"(t=1,dataPL=1)} + \#\_days_{dataPL} * ESP\_Pumping_{esp,dataPL,t} * ESP\_Efficiency_{esp} \\ - \#\_days_{dataPL} * Generation_{esp,dataPL,t} \quad (1.13)$$

$$\forall esp \in Ele, \forall dataPL \in Kal, t \in T \\ ESP\_Pumping_{esp,dataPL,t} + Generation_{esp,dataPL,t} \\ \leq ESP\_CpctyLoading_{esp} * InOper_{esp,dataPL} * (1 - DTable\_OwnCons_{esp}) \\ * (1 - DTable\_Losses_{dataPL,esp}) \quad (1.14)$$

$$\forall esp \in Ele, \forall dataPL \in Kal, t \in T \\ ESP\_Balance_{esp,dataPL,t} \\ \leq \#\_days_{dataPL} * ESP\_WaterCapacity_{esp} \\ * InOper_{esp,dataPL} \quad (1.15)$$

Transmission capacity constraints of cross-border lines determine introduction of equations constraining the values of international flows. The cross-border flows present in the capacity budget equations (1.16 and 1.17) are thus constrained for both import and export in the following way:

$$Flows_{ct,dataPL,t} \leq DTable\_CBF\_NTC_{ct,PL',dataPL,t} \quad \forall ct \in CT, \forall ct\_t \in CT, \forall dataPL \in Kal, t \in T \quad (1.16)$$

$$Flows_{ct,dataPL,t} \geq DTable\_CBF\_NTC_{PL',ct\_t,dataPL,t} \quad \forall ct \in CT, \forall ct\_t \in CT, \forall dataPL \in Kal, t \in T \quad (1.17)$$

To reflect the operation of the electricity system more precisely, a simulation was performed of TSO's actions aimed at assuring secure operation of the National Electricity System by introducing the equation (1.18) that defines the minimum activated capacity volume in dispatchable units on a given day. The 1.12 constant is the equivalent of the  $A^h$  parameter in the Transmission Grid Code (IRiESP).

$$\sum_{es} [InOper_{es,dataPL} (1 - DTable\_OwnCons_{es}) * (1 - DTable\_Losses_{dataPL,es})] \\ \geq 1.12 \left( DTable\_Demand_{dataPL,t} - \sum_{ens} (DTable\_Cpcty_{ens} * DTable\_Production\_ENS_{dataPL,t,ens}) - \sum_{kons} DSR_{kons,dataPL,t} \right) \quad (1.18)$$

$$\forall es \in Ele, \forall dataPL \in Kal, t \in T, kons \in Kons$$



## List of values used throughout the model

### Sets

Name	Dimension	Description
dataPL	(*)	Days
ele	(*)	All power plants
ens	(ele)	Non-dispatchable power plants
eres	(ele)	RES power plants
es	(ele)	Dispatchable power plants
esp	(ele)	Pumped-storage power plants
kons	(*)	Consumers
olm	(*)	Opencast lignite mines
map_olm_ele	(olm,ele)	Power station to mine mapping
own	(*)	Owners
s	(*)	Scenarios
t	(*)	Hours
yf	(*)	All years
y	(yf)	Years in simulation
ct	(*)	Set of countries
ct_t	(ct)	Auxiliary set of countries

### Parameters

Name	Dimension	Unit	Description
DTable_CAPEX	(ele)	PLN/MW	CAPEX per MW (PLN)
DTable_Efficiency	(ele)	%	Power plant net efficiency (%)
DTable_Own_Cons	(ele)	%	Mean own consumption factor for CDGUs (% of net output)
DTable_REStarget		%	Target RES share in electricity demand (%)
DTable_DSR_Price	(kons)	PLN/MWh	DSR bided price (PLN per MWh)
DTable_DSR_Cpcty	(kons)	MW	DSR bided capacity (MW)
DTable_Gen_Cost	(dataPL, ele)	PLN/MWh	Variable cost of production in given power plant on given day
DTable_CostActvtn	(dataPL, ele)	PLN/MW/u	Activation costs (PLN per MW per activation)
DTable_LF	(ele)	%	Load-factor for forced operation (% activated net capacity)
DTable_LimitOLM	(olm)	TJ	Lignite mines' production output limit (tonnes * calorific value = TJ)
DTable_Cpcty	(ele)	MW	CDGUs' attainable capacity in a given period (MW gross) (in case of capacity planning – attainable capacity limit)
DTable_OM	(ele)	PLN/MW	Annual O&M expenses (PLN per MW)
DTable_Production_ENS	(dataPL, t, ele)	%	Hourly average production from non-dispatchable sources (% of attainable capacity)
ESP_Efficiency	(ele)	%	Pumped-storage plant efficiency (% of energy stored that can later be fed to the grid)
ESP_WaterCapacity	(ele)	MWh	Maximum reservoir capacity (MWh that may be produced from full reservoir defined as capacity to generation capacity ratio)
ESP_Cpcty_Loading	(ele)	%	Pumped-storage plant pumping capacity (pumping capacity to generation capacity ratio)
DTable_CBF_Price	(ct, dataPL, t)	PLN/MWh	Price in a given country PLN/MWh
DTable_CBF_NTC	(ct, ct_t, dataPL, t)	MW	NTC for connection from country A to country B (MW)
DTable_TechMin	(ele)	%	Stable technical operation minimum (% of attainable capacity)
DTable_Losses	(dataPL, ele)	%	Total in-plant capacity losses (% of attainable capacity)
DTable_Demand	(dataPL, t)	MW	Hourly average capacity demand in the National Electricity System (MW net)
#_days	(dataPL)	-	Number of day types (counter)



## Variables

Name	Dimension	Unit	Description
<b>Positive variables</b>			
Generation	(ele, dataPL, t)	<i>MWh</i>	Net generation of dispatchable power plants (MWh)
Cpcty	(ele)	<i>MW</i>	Gross installed capacity of generating unit (MW)
InOper	(ele, dataPL)	<i>MW</i>	Active gross capacities in power plants (MW)
Activtn	(ele, dataPL)	<i>MW</i>	Gross capacities being activated in power plants (MW)
ESP_Balance	(ele, dataPL, t)	<i>MWh</i>	Loading balance for pumped-storage plants (MWh)
ESP_Pumping	(ele, dataPL, t)	<i>MWh</i>	Pumped-storage plant pumping (MWh)
DSR	(kons, dataPL, t)	<i>MWh</i>	DSR activated capacity (MWh)
<b>Free type variables</b>			
Flow	(ct, dataPL, t)	<i>MWh</i>	Balance of flows with other countries (MWh) (-export +import)
Z		<i>PLN</i>	Goal function – total system operating cost (PLN)

## Equations and inequalities

No.	Name	Dimension	Unit	Description
1.1	Funkcja_celu	(*)	<i>PLN</i>	Goal function reflects the sum of total discounted system costs
1.2	eqBilansMocy	(dataPL, t)	<i>MW</i>	Equation balancing demand and supply
1.3	eqOgrOZE	(*)	<i>MWh</i>	Constraint on minimum RES generation
1.4	eqOgrGenerSter	(es, dataPL, t)	<i>MWh</i>	Upper constraint of dispatchable power plant generation
1.5	eqGenerNieSter	(ens, dataPL, t)	<i>MWh</i>	Generation from non-dispatchable power plants
1.6	eqOgrMocEle	(ele)	<i>MW</i>	Allocation of installed capacities to existing power plants
1.7	eqOgrMocSter	(es, dataPL)	<i>MW</i>	Upper constraint of variable <i>InOper<sub>es,dataPL</sub></i>
1.8	eqOgrDSR	(kons, dataPL, t)	<i>MWh</i>	Constraint on value of DSR
1.9	eqOgrGener	(es, dataPL, t)	<i>MWh</i>	Lower constraint of generation in dispatchable power stations
1.10	eqOgrMocUruch	(es, dataPL)	<i>MW</i>	Upper constraint of capacities being activated
1.11	eqOgrLF	(es, dataPL, t)	<i>MWh</i>	Lower constraint of generation in dispatchable power plants (not lower than average <i>Load Factor</i> )
1.12	eqOgrEnerWB	(olm)	<i>TJ</i>	Constraint of production capacity of lignite opencast mines
1.13	eqBilansSpomp	(ele, dataPL, t)	<i>MWh</i>	Balancing equation of pumped-storage peaking plant
1.14	eqOgrPomp	(esp, dataPL, t)	<i>MWh</i>	Constraint of electricity consumed for pumping at a pumped-storage peaking plant
1.15	eqOgrPoizbior	(esp, dataPL, t)	<i>MWh</i>	Constraint resulting out of pumped-storage peaking plant reservoir capacity
1.16	eqOgrMocImport	(ct, ct_t, dataPL, t)	<i>MW</i>	Constraint of cross-border flows in electricity import to Poland
1.17	eqOgrMocEksport	(ct, ct_t, dataPL, t)	<i>MW</i>	Constraint of cross-border flows in electricity from Poland
1.18	eqOgrMinWolumen	(dataPL, t)	<i>MW</i>	Constraint of the minimum capacity volume in dispatchable power plants resulting from TSO's actions (Transmission Grid Code)

## Simulations for electricity market modelling

Modelling an electricity market requires making assumptions on behaviour of the demand side and the supply side. For this purpose a scenario analysis was constructed based on simulations of various values including:

- Demand
- Onshore wind generation
- Offshore wind generation
- PV generation
- Biogas plant generation
- Run of the river hydrogeneration
- Biomass plant generation
- Combined heat and power plant generation
- Cross-border exchange
- CDGU capacity outages

The analysis is based on presentation of scenarios of hourly values of the above variables; scenarios were grouped by weather conditions where the grouping variable was the barometric pressure.

Data for 2012-01-01 – 2015-12-31 was used, in particular including:

- Hourly demand in National Electricity System – data from PSE
- Onshore wind generation – PSE
- PV generation – data from Germany (50Hz, Tennet, Transnet, Amprion)
- CHP generation – PSE
- Cross-border exchange – PSE
- CDGU capacity outages – PSE

For other values fixed assumptions were adopted leaving a potential for development by making the approach more detailed. The collected data was divided into 18 groups (6 day types x 3 weather groups [broken down by barometric pressure]). The CDGU capacity outages were additionally broken down by fuel type. Then 1000 observations for each group were randomly drawn with replacement in uniform distribution.

Due to the sampling scheme it is natural to base the scenarios on weather data yielding a preliminary breakdown into low pressure, transition (weather front) and high pressure scenarios. Subsequent scenarios were set based on combinations of results of simulations from historical data.

Remarks on random drawing method:

a) Power plant outages are different in different scenarios, but symmetrical for all units. For example: unit A in scenario 1 has chance for a 10% outage, unit B has 20%. In scenario 2 unit A has a chance for a 5% outage, unit B has 10%. This is necessary since if the model is to decide whether it is profitable to keep the unit in the market or not, then if there are scenarios in which the unit is not available at all (and at the same time these are those extreme scenarios that would justify its existence), then the model would lose such information.

b) Random drawing of scenarios for historical years takes place according to empirical distribution of probability, and probability of scenarios is calculated according to historical values. For future years the climate changes are presented in such way that probability of extreme scenarios is modified according to forecasts by external institutions.

c) The randomly drawn scenarios are mixed with respect to low, transmission, high pressure weather combinations in subsequent types of days reflecting seasons in such way as to allow effective use of model's functionalities referring to annual values (share of RES and lignite production).



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